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On a New Mode of Description in Physics

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Abstract

We explore the possibilities of a new informal language, applicable to the microdomain, which enables such characteristics as superposition and discreteness to be introduced without recourse to the quantum algorithm. In terms of new notions that are introduced (e.g. 'potentiation' and 'ensemblation'), we show that an experiment need no longer be thought of as a procedure designed to investigate a property of a 'separately existing system'. Thus, the necessity of a sharp separation between the 'system under observation' and the 'apparatus' is avoided. Although the new language is very different from that of classical physics, classical notions appear as a special limiting case.

This new informal language leads to a mathematical formalism which employs the descriptive terms of a cohomology theory with values in the integers. Thus our theory is not based on the use of a space-time description, continuous or otherwise. In the appropriate limit, the mathematical formalism contains certain features similar to those of classical field theories. It is therefore suggested that all the field equations of physics can be re-expressed in terms of our theory in a way that is independent of their space-time description. This point is illustrated by Maxwell's equations, which are understood in terms of cohomology on a discrete complex. In this description, the electromagnetic four-vector potential and the four-current can be discussed in terms of an 'ensemblation' of discontinuous hypersurfaces or varieties. Since the cohomology is defined on the integers the charge is naturally discrete.

1. *Introduction*

Various forms of contemporary relativistic quantum field theories and the closely related *S*-matrix approach make the explicit assumption that all the physically relevant properties may be obtained by ordering the fields or scattering amplitudes on four-dimensional differential manifolds. The success that has been achieved using this assumption, particularly, for example, in quantum electrodynamics, has created the feeling that these theories or, at least, closely related theories are essentially correct. This has led to a considerable effort to search for better mathematical techniques with which to overcome the outstanding problems.

We recognise the value of the work done along these lines, but we wish to point out that a description based on a differential manifold has serious limitations. Indeed, various attempts have already been made to investigate the precise nature of these limitations (Atkinson & Halpern, 1967; Hill, 1955; Schild, 1949; Snyder, 1947). Some of these enquiries have considered in what way non-usual topologies affect the results, while others have attempted to remove the need for renormalization by adopting a discrete topology, usually by introducing some form of fundamental length. However, these attempts mainly investigate the effects of various assumptions on the *formal* aspects of the theory. Indeed, there is a widespread feeling that a change in the informal language (i.e. that used to describe the experimental situation) is neither necessary nor even possible (Bohr, 1934, 1958). This means that, once again, there is a heavy emphasis on the mathematics.

It is our view that it is not sufficient to change only the formalism and that, as has already been argued elsewhere, a more radical approach is needed (Bohm, 1968; Hiley, 1968). New informal languages and their extensions into mathematical forms need to be investigated.

In any new approach that is relevant to physics, discreteness should appear as a natural consequence of the informal considerations and should not be arbitrarily imposed. Thus it is not possible to obtain discreteness *naturally* if the classical notions of particle, trajectory, potential, field, etc., continue to be taken as *primitive* concepts since these notions were developed specifically for the continuum. Of course, it is necessary for such notions to emerge at some more abstract level as a result of, say, some form of suitable statistical averaging procedure. However, in this way the classical forms will arise as a consequence of some deeper, more primitive theory.

In this paper we consider a radically new theory that uses a novel informal language which is very different from that usually adopted in physics today. In contrast to our present way of using language, which places emphasis on *separate* objects in interaction, we give primary relevance to *activity* and *wholeness* in the sense of undivided movement.

Our main reason for emphasising these notions is that they are implicit in quantum theory. For example, the indivisibility of the quantum of action implies a merging of the 'observed system' and the 'observing apparatus' so that the two are inseparable and, therefore, constitute a whole in which analysis into parts is not relevant. This whole flows and merges into the totality of the universe, including the human observer.

In exploring new informal languages, we have been guided by our experience with quantum theory. For instance, each 'quantum state' implies potentialities (Bohm, 1960) whose realisation can be incompatible in the sense that different realisations require mutually exclusive experimental arrangements. In a description which uses the informal language of classical physics, this incompatibility is understood in terms of the Principle of Complementarity. In our new informal language we do not need any such principle. However, we do need new terms in order to call attention to a different way of thinking. We thus introduce a new basic term 'potentiation'. The full meaning of this term will be discussed in Sections 2 and 3 but, for the present, we can regard it as meaning the realisation of potentialities.

In terms of potentiation, an experiment has a new meaning (see Section 3) In classical physics the results always refer to the properties of a system existing separately from the observing apparatus. In our description there is no separately existing system. The overall experimental arrangement potentiates a content, the meaning of which depends explicitly on this arrangement. Thus there is no separation of a 'system' under observation from an 'observing apparatus'; and this is just what is implied by the finite nature of the quantum of action.

Further, from quantum theory, we know that the potentialities are not discussed in terms of individual events but in terms of ensemble averages. Again, in our new informal language we introduce another basic term, namely, 'ensemblation' which essentially means the formation of ensembles. As will be seen in Section 2, these ensembles are not only of the type encountered in statistical mechanics but can also be of a very different nature.

Thus the essential features of the quantum theory are contained in the notions of discreteness, potentiation and ensemblation. These features arise naturally in a certain mathematical description which makes use of the descriptive terms of a cohomology theory with values in the integers (Cairns, 1958). This theory also contains a linear superposition principle analogous to that of the quantum theory (see Section 3). Our superposition principle differs from that of the quantum theory, however, in the sense that it cannot be expressed in terms of a Hilbert space (except as an approximation valid in a suitable limiting case). Indeed, it is because of this that we can describe the experiment in a different way which does not give fundamental relevance to the Principle of Complementarity or to the Uncertainty Principle.

As a matter of fact, we show that in terms of our new mode of description, the 'wholeness' of the 'observing instrument' and the 'observed content' is just as relevant in the classical domain as it is in the context of quantum theory. Thus, to illustrate, we consider as a particular example, the expression of the laws of classical electrodynamics in terms of cohomology theory. Indeed, Misner & Wheeler (1957) have already indicated that once the differential equations are written in terms of differential forms and exterior derivatives, these equations can be re-interpreted as defining a de Rham cohomology. In this way, they have been led to propose that (continuously variable) charge can be explained as a certain topological aspect of space-time, i.e. a 'worm hole' in which the electromagnetic field is 'trapped'. We suggest instead, however, that the de Rham cohomology can be understood, in the light of our theory, as defining a cohomology on abstract simplicial complexes with values in the integers. In this way, the equations can be given a meaning that is independent of whether or not there is an underlying space-time description, and of whether the structures involved are continuous or discrete. In our illustration, in terms of the laws of classical electrodynamics, we are thus led to propose a new meaning for the electromagnetic four-potential and for the four-current, in which charge is naturally discrete.

The detailed contents of this paper are as follows. In Section 2 we introduce and discuss the new basic notions, while in Section 3 we propose a mathematical description of potentiation and ensemblation. In Section 4 we consider as a particular example the expression of the laws of classical electrodynamics in terms of cohomology theory, and show how our description leads to a discrete charge.

2. *Informal and Formal Languages in Physics*

Before the development of the quantum theory, the fundamental descriptive language of physics contained the assumption, either implicit or explicit, that the world is constituted of separately existing objects, i.e., 'things in themselves', which interact with each other according to well-defined laws. Bohr was probably the first to realise that the finite nature of the quantum of action implies that this description cannot be relevant in the quantum context. Thus the separation of the 'observed object' from the 'observing apparatus' is no longer a tenable form of description and this means that 'observed properties' cannot consistently be attributed to the 'object' alone. Originally, it was implied that this situation is the outcome of an unknown disturbance of the 'observed system' in its interaction with the 'observing apparatus'. But, as was pointed out, especially by Bohr, the implications of the quantum of action go much further than this, and indeed call into question the entire notion of an 'object in itself' as a relevant form of description of physics.

However, along with most other physicists, Bohr maintained that ordinary common-sense language, refined where necessary with the language and the concepts of classical physics, has to be used in the description of the experiment, and that any other form of description of the experiment is impossible. In this way, there was established a sharp separation between the language for describing experiments and the formal mathematical language used for making theoretical inferences about the results of the experiment. To be sure, a certain relationship of correspondence between these two kinds of language was indicated. But the formal mathematical terms (i.e. Hilbert space, operators, commutators, etc.) were regarded as having no relevance for discussing the experiment itself.

The informal language relating theory and experiment (i.e., probabilities, scattering amplitudes, etc.) was thus regarded as essentially determined and unchangeable, even though the formalisms, of course, underwent quite considerable alterations (e.g. renormalisation, *S*-matrices, Regge pole theory, etc.). Indeed, there appears to have arisen a widespread tacit agreement that basic advances would now have to be made in the mathematics alone, while the general informal language relating the experiment to theory would continue, more or less as it had been since it was originally introduced in connection with Schrodinger's equation.

This approach still, however, treats the *results* of an experiment as an 'object of discourse' which can be abstracted from the overall experimental arrangements and made the subject of mathematical inferences, which ultimately refer to 'laws of nature' that would be independent of, and separate from, the apparatus. In a sense, therefore, the 'thing in itself' is still relevant but at some more abstract level. However, to hold onto the 'thing in itself' in this way is still not consistent with the full implications of the quantum of action. To go beyond the customary mode of description which leads to this inconsistency, we have to question the assumption that ordinary common-sense language (refined with the aid of classical concepts) is the only possible one for discussing the experiment.

In particular, it is necessary to go outside of what is generally regarded as the domain of physics and to enquire into our perceptions, from which our knowledge of objects must eventually come. Is the notion of an object actually basic in perception, or is it that we have come to regard such a notion as self-evident, and therefore fundamental, because of environmental conditioning and training?

There is a great deal of experimental evidence coming from psychological and neurological investigations that, beginning from early childhood, we actually learn to abstract the notion of an object from a more fundamental level of perception (Bohm, 1965b). What is primitive is perception of movement, or of change, or of a break in some regular order or arrangement. From an ensemble of such perceptions of movement, something relatively invariant is abstracted, and this abstraction is the foundation for the presentation of perception in the form of relatively fixed or slowly moving objects. This is indeed very similar to what happens in relativity in which, likewise, the 'object' is abstracted from invariants of movement (Bohm, 1965a).

We are thus led to suggest that primitive perception is close, in a certain sense, to the most advanced developments of physics, whereas classical physics, or 'common-sense' descriptions, are high-level abstractions which we have learned to regard as fundamental because of an extensive process of conditioning. Thus in primitive perceptions, the 'thing in itself' is not fundamentally relevant and the same holds in quantum theory and in relativity.

However, because our general language has been developed to meet certain everyday needs with regard to the use of objects, the noun, which is the indication of such an object, has been given a fundamental role, while the verb, which calls attention to action, tends to have a secondary importance. Therefore, to cease to take the 'thing in itself' as primitive, we will instead give a basic role to the verb (while nouns will be regarded as abstractions from verbs). This approach emphasises movement and activity and implies that objects are, even in informal discourse, to be regarded as relative invariants of such movement and activity. Thus, the 'object in itself' is no longer taken as a basic term of description. Since movements can generally flow into each other and merge, the division of the world into separate constituents has also been dropped. Like the 'object', it will arise as an abstraction from an unbroken and undivided totality of movement, which we call *holomovement* (using the Greek prefix 'holo', which means 'whole').

In a given context, certain aspects of such a totality will be relevant, while others will not. It is useful here to bring back the word 'to relevate', which has dropped out of common usage. This means 'to lift into attention (e.g. as in 'relief'). In any perception, certain aspects can thus be said to be relevated.

We may extend the usage of this word to say that, in a certain sense, a given experimental arrangement also relevates a content. In accordance with our discussion in the introduction, this arrangement actively helps to create conditions for a particular phenomenon to appear in a particular form and thus to stand out as being relevant. By using the verbal form, 'to potentiate', we emphasise this active role (and, of course, we cease to regard the 'thing in itself' as a fundamental descriptive term).

However, we do not wish to imply that only an experimental apparatus can relevate and potentiate a content. On the contrary, we propose that any arrangement of matter potentiates and relevates a certain content, and that the action of the apparatus is thus a special case of this, in which the outcome is particularly simple to interpret. Since the observing apparatus is not given a special role in the description, it follows that the subjective observer also has no special role

As pointed out in the introduction, the potentiated content can generally be described as an ensemblation. In this connection, it is significant to note that the definition of 'ensemble' given in the dictionary is that each member is related only to a whole. This feature of an ensemble can be illustrated by considering a painting. The individual spots of paint can be said to ensemblate, to form a whole content, including trees, houses, etc. This whole content is evidently of a very different character from the individual spots of paint, whose only significant relation is that they form such a whole. Similarly, movements ensemblate to form wholes. Thus in perception, all the changes or breaks in movement ensemblate to give rise to the relatively invariant objects.

In physics, we not only have statistical ensemblations but also ensemblations of a more general kind. For example, in bubble chambers, we see a sequence of dots which form an irregular curve that is interpreted as the track of a particle. This irregular curve can be described as an ensemblation of a certain number of bubbles. In addition, these bubbles may be further ensemblated to a 'smooth curve', which is a sort of average 'track' and, in turn, a number of these 'tracks' ensemblate to form what may be called 'a whole picture', describable mathematically in terms of wave functions or 'quantum states'. Such wave functions are as different in character from the bubbles as the content of a painting is from the spots of paint.

As indicated earlier, the further developments of the mathematical description of the 'whole picture' (e.g. field theories, S-matrices, etc.) are limited by the 'classical' informal language currently used for describing the experiment. Because our own informal language, as discussed above, is different in that it emphasises movement and wholeness, we can introduce new forms of mathematics going outside the limits of theories that can be interpreted in terms of a 'classical' informal description. Thus, for example, our descriptions in terms of simplicial complexes, using a cohomology over the integers (see next section), goes beyond the notion of a Hilbert space. The entire scheme of unitary transformations with its probability interpretation is no longer fundamentally relevant (except in suitable limited cases). Moreover, quite new directions of enquiry are opened up, which we shall explore in later papers.

3. *Basic Mathematical Description of Ensemblation*

In the previous section we have introduced some new informal terms, i.e. potentiation and ensemblation, which are relevant in physics. We now introduce a mathematical formalism in which the potentiations and ensemblations can be given a more articulated and detailed description.

The very notion of potentiation is essentially non-local so that a mathematical theory using continuous coordinates is inappropriate (as will become apparent when we attempt to relate our theory to the conventional ones). Fortunately in combinatorial topology, a mathematical theory which does not depend on locality has already been developed and appears very suitable for the detailed description of our ensemblations. This is the theory of homology and cohomology (Hilton & Wylie, 1960).

We use the terms of homology theory as basic descriptive forms. We begin by calling each potentiation an abstract 0-simplex. Then if two potentiations, A and B, are related in some way, we will represent this relation by a 'connection' or 'line' and call the relation $A \circ B$, an abstract 1-simplex. In our later work we will find it convenient to distinguish between the relations $A \circ B$ and $B \circ A$. This can be achieved by attaching a 'direction' or 'orientation' to the abstract simplex. A cyclic relationship between three potentiations $A \circ B \circ C$ will be represented by an abstract oriented 2-simplex and so on. (For convenience, we will simply use the term simplex and understand it to mean an abstract oriented simplex.) Thus the potentiations and their relationships are said to form a simplicial complex which can be used to give a detailed description of a totality of potentiations.

We can form an ensemblation from such potentiations in a variety of ways. For example, we can relevate the various potentiations by weighting them with suitable integer coefficients. Or we can relevate the various relationships between different potentiations by a similar weighting of simplexes of higher dimension. When the integers are thus used as weights, the relevant ensemblations are equivalent to what are called in topology, integral chains on the complex. The dimensionality of the chains will depend on whether we are considering the 0-simplexes, the 1-simplexes, etc. (e.g. a 1-chain will be a linear combination of 1-simplexes suitably weighted).

In this ensemblation we can now use what are called the boundary operators to find the subset of chains which are cycles (i.e. which have no boundary) and the subset of cycles which are bounding cycles (i.e. those cycles which are boundaries of chains). In this way we can discuss the homology properties of the ensemblation.

We propose to describe the results of an experiment as an ensemblation of chains and cycles on a simplicial complex of potentiations. Thus the results are actively potentiated in the holomovement and are not treated as 'things in themselves'. The holomovement, however, involves the apparatus and the general background, along with the results, in an inseparable way. And, as indicated in earlier sections, this implies that we have to use the same informal and formal languages to describe the apparatus as we use for discussing the results of experiments, along with the inferences to be drawn from these results.

The form and structure, as well as the activity of the apparatus can be described in terms of a dual complex which we shall call the complex of copotentiations. What is relevant for the theoretical inferences is a certain relationship of potentiations and copotentiations which is mathematically termed their 'intersection' and which is invariant to the changes of the basis of the simplicial description. Thus, as far as the theory is concerned, the 'common sense' language of classical physics no longer plays a fundamental role in the description.

Of course, the distinction between a potentiation and copotentiation is merely a convenient form of description and is not to be taken as implying their separate existence. Indeed, what are taken as copotentiations at one level can be regarded as potentiations on another level (e.g. the copotentiations describing the apparatus can be expressed as potentiations on a 'finer mesh'). Thus ultimately our approach implies a hierarchy of complexes, though in any given context a limited number of steps in the hierarchy will be adequate.

If we regard the dual of a complex as a functional, we can use the descriptive terms of cohomology theory for the copotentiations. By using integral weights for the copotentiations and for the relationships between them, we obtain the corresponding cochains and cocycles.

The ensemblations, which are to be regarded as the physically relevant content, will (as has already been pointed out) be described in terms of the intersection of the relevant chains and cycles with the corresponding cochains and cocycles. That is, if the relevant physical situation is described by sets of p-chains, $C^j_{(p)}$, and by sets of p-cochains, $C^{(p)}_k$, then the intersection is written as

$$(C^{(p)}_k, C^j_{(p)}) = g_k{}^j \tag{3.1}$$

where $g_k{}^j$ is an integer. In fact, each situation will be described by matrices with integer elements which will be called *intersection matrices*. Thus our theory contains a type of discreteness which seems to be called for in quantum theory and in other physical theories (e.g. discrete charge in electrodynamics).

A further important aspect of this general type of description is that it contains a superposition principle analogous to the one used in quantum theory. This is because the chains and cycles, as well as the cochains and cocycles, can be added with integer coefficients to form an Abelian group. Or, to put it less formally, given any two physically relevant chains $C^1_{(p)}$ and $C^2_{(p)}$, then if α and β are integers, the chain

$$C^3_{(p)} = \alpha C^1_{(p)} + \beta C^2_{(p)} \tag{3.2}$$

is also a physically relevant chain. This is evidently similar to the superposition principles that are used not only in quantum theory, but also in other branches of physics. The key difference is that the coefficients are restricted to integers (so that, for example, we are not dealing with a Hilbert space). However, in the limit when the integers are very large, it is clear that our chains correspond in some approximate sense to vectors and tensors in Hilbert space.

Because there is a physical meaning to linear superposition, we can now define linear transformations of the chains and cochains which are similar to the unitary transformations in quantum theory but which differ in important ways as well. Thus a linear transformation on the p-chains can be written as

$$C'^j_{(p)} = a_i{}^j\, C^i_{(p)} \tag{3.3}$$

while the corresponding adjoint transformations on the cochains is

$$C'^{(p)}_k = C^{(p)}_m\, b_k{}^m \tag{3.4}$$

The intersection matrix then transforms bilinearly in the following way

$$\mathbf{g}' = \mathbf{agb} \tag{3.5}$$

A further analogy with quantum theory is now evident. For the 'observables' of quantum theory, which are represented by matrices A_{ij}, undergo a similar bilinear transformation under change of basis

$$\mathbf{A}' = \mathbf{S}^\dagger\,\mathbf{AS} \tag{3.6}$$

In our theory, the physically relevant 'observable' will, of course, be described in terms of intersection matrices rather than in terms of usual quantum mechanical matrices which represent operators in a Hilbert space.

Once the implications of our approach have been understood, it is not just a simple matter of looking at the various experiments that have already been performed and trying to understand them in terms of the present scheme. Many experiments are designed to test specific questions as raised in the particular form used in a given theory. For example, in classical physics there are questions about particle orbits, causal ordering by means of fields, etc., while in the quantum domain there are questions about energy levels and about probabilities of various processes. Our new description is radically different and therefore the old experiments may not ask questions that are appropriate to it.

The descriptive form that we have introduced has to be taken further before we can understand exactly how a particular apparatus can be described in terms of a particular set of copotentiations. We leave this question on one side and will take it up again in a later paper. However, in the next section of the present paper we shall consider an illustrative classical example in which a similar question arises, but in a simpler way.

4. *The Laws of Electrodynamics Described in Terms of Our New Language*

The laws of electrodynamics were first expressed in terms of integrals of fields over cycles of varying dimensionality, e.g. Ampere's law, Faraday's law, Gauss's law, etc. It is only from the extrapolation of these integral laws to infinitely small cycles that one obtains Maxwell's equations. Thus these equations go considerably beyond what can be inferred from observations alone. The relative ease of the mathematical application of the differential form of Maxwell's equations has made this approach attractive. However, the infinities which arise in the indefinite extension of this form, both classically and quantum mechanically, imply that it may be appropriate to go back to the integral form in spite of the possibility of greater mathematical difficulty. The appropriate mathematics for doing this is just the theory of complexes of chains and cochains that we have described earlier.

In this paper, we will restrict ourselves to classical electrodynamics. In our description, the relevance of the wholeness of the instruments and the observed content is evident even in the classical context. For every observation is the integral of a field quantity over a cycle. The field is a potentiality of 'empty space'. Moreover, the cycles are also potentialities in the sense that the cycle which is physically relevant in any given situation will depend on the experimental arrangement. Remembering that the role of the chain and cochain is conventional, so that the two can be interchanged, we find it convenient to describe the fields in terms of cochains, while the cycles are described in terms of chains. The intersection matrices of chains and cochains (which in ordinary terminology would be the integral of fields over cycles) is then the physically relevant quantity in terms of which the laws of physics are to be expressed. We are thus led to a kind of 'wholeness' very similar to that arising in the context of quantum theory.

In order to help visualise what is meant, we will describe the fields and the cycles in terms of the usual space-time description involving vectors, tensors, etc. (but recalling that this is a particular case of our general description in terms of chains and cochains). We begin with the vector potential A_μ. Usually this is taken as a continuous function but we are going to regard it as a discontinuous one which resembles a set of δ-functions in the sense

$$\int_{C_{(1)}} A_\mu dx^\mu = n \tag{4.1}$$

where n is an integer. One may visualise this by thinking that in any given region there is an array of surfaces, e.g., hyperplanes, at which the vector potential undergoes a discontinuous change such that as the 1-chain, $C_{(1)}$, crosses the surface, the integral increases by unity. In other words, n counts the number of planes crossed by $C_{(1)}$.

Let us now consider the meaning of Stokes' theorem,

$$\int_{BC_{(2)}} A_\mu dx^\mu = \int_{C_{(2)}} \partial_\nu A_\mu dx^\nu \wedge dx^\mu \tag{4.2}$$

where $C_{(2)}$ is a two-dimensional area and $BC_{(2)}$ is, by definition, the boundary of this area. If one thinks of the meaning of the right-hand side, one can see that the integral can fail to be zero only if some of the surfaces corresponding to A_μ have boundaries which cross the area $C_{(2)}$. To make the example yet simpler, consider the projection of the vectors and tensors into a three-dimensional sub-space; then the meaning of Stokes' theorem is that the number of surfaces having boundary lines which cross $C_{(2)}$ is equal to the number of surfaces crossed by the boundary of $C_{(2)}$. This means that in a certain sense it may be said that $\partial_\nu \wedge A_\mu$ corresponds to the boundary of the surfaces described by A_μ. This kind of correspondence can be extended to any number of dimensions.

As an example, the electromagnetic field tensor, $F_{\mu\nu}$ will correspond to a two-dimensional variety and $\partial_\alpha \wedge F_{\mu\nu}$ corresponds to the boundary of this variety which is one-dimensional. Thus Maxwell's first equation, $\partial_\alpha \wedge F_{\mu\nu} = 0$, implies that the variety corresponding to $F_{\mu\nu}$ is closed, i.e., it has no boundary. If the topology is 'trivial' this means that $F_{\mu\nu} = \partial_\nu \wedge A_\mu$. Or, in other words, the $F_{\mu\nu}$ variety is the boundary of the hyperplanes corresponding to A_μ. If we restrict ourselves to a three-dimensional sub-space, the $F_{\mu\nu}$ will correspond to discrete lines of force for the fields. In the limit, where the varieties are 'very dense', we will get a quasi-continuous description.

We now put the above results in terms of our own language of chains and cochains so that the laws will be expressed independently of any underlying continuous space-time. The basic physically relevant quantity $\int_A F_{\mu\nu} dx^\mu \wedge dx^\nu$ corresponding to the integral of the field over the area A is now to be written as

$$(f^{(2)}, C_{(2)}) = n \tag{4.3}$$

where n is an integer, $C_{(2)}$ is a 2-chain corresponding to the cycle of integration and $f^{(2)}$ is the 2-cochain corresponding to the field. Maxwell's first equation,

$$\int_{C_{(3)}} \partial_\alpha F_{\mu\nu}\, dx^\alpha \wedge dx^\mu \wedge dx^\nu = \int_{BC_{(3)}} F_{\mu\nu}\, dx^\mu \wedge dx^\nu = 0 \tag{4.4}$$

now corresponds to

$$(Bf^{(2)}, C_{(3)}) = (f^{(2)}, BC_{(3)}) = 0 \tag{4.5}$$

Here B is the boundary operator. The meaning, for example, of $BC_{(2)}$ is that one is to replace the 2-chain $C_{(2)}$ by that 1-chain which is its boundary. Thus we obtain laws in which differential operators are no longer used and in which assumptions of continuity are not needed.

Maxwell's second equation requires the definition of the dual tensor $F^*_{\mu\nu}$. In continuous geometry this is done with the aid of a metric $F^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} g^{\alpha\lambda} g^{\beta\kappa} F_{\lambda\kappa}$. The dual to any element is essentially a kind of perpendicular or normal. Thus the dual to A_μ is $\epsilon_{\mu\lambda} g^{\lambda\nu} A_\nu$ which corresponds to an ensemble of lines that is perpendicular to the surfaces defined by A_μ. In terms of topological notions, however, there is no meaning to perpendicularity. Rather, the dual has to be defined in a different way which does not require the metric tensor in any basic sense. In later papers we shall discuss how this is to be done. However, for the present, we shall accept that there is a suitable dual complex and see the meaning of Maxwell's second equation as applied to this dual.

Beginning with space-time notions, we write

$$\partial_\lambda \wedge F^*_{\mu\nu} = \epsilon_{\lambda\mu\nu\alpha} j^\alpha \tag{4.6}$$

where j^α is the current density. In integral form this becomes

$$\int_{C_{(3)}} \partial_\lambda \wedge F^*_{\mu\nu}\, dx^\lambda \wedge dx^\mu \wedge dx^\nu = \int_{C_{(3)}} \epsilon_{\lambda\mu\nu\alpha} j^\alpha\, dx^\lambda \wedge dx^\mu \wedge dx^\nu \tag{4.7}$$

To visualise this we say that the lines of current correspond to the boundaries of the $F^*_{\mu\nu}$ varieties. Or, to simplify, let us take a three-dimensional subspace. The $F^*_{\mu\nu}$ will now correspond to lines of force and the charges to their boundaries. Thus charge will come out naturally as discrete. Maxwell's second equation thus means that the total charge inside a given region is equal to the number of lines of force whose boundaries are in that region.

We now return to the consideration of the full four-dimensional space-time. In terms of chains, Maxwell's second equation can be written as

$$(B^* f^{(2)}, C_{(3)}) = (j^{(2)}, C_{(3)}) \tag{4.8}$$

Thus the current density corresponds to an abstract 1-chain which is the boundary of a 2-chain.

This completes the expression of the laws of classical electrodynamics in terms of chains and cochains.

5. *Conclusions*

We have introduced a new form of description in which wholeness, activity and discreteness are given fundamental relevance. Both quantum mechanically and classically, the division between the observing instruments and the observed content no longer arises in our description. The same general form of language can be used both to describe the experiments and to make theoretical inferences from them.

Formally, the use of chains and cochains of simplicial complexes is a natural extension of the informal language from which we started. Using this form of mathematics we showed how one could describe an experiment, quantum mechanically or classically, in terms that are not basically different from those needed in the description of the 'observed results'. In particular, as an illustrative example, we put the laws of classical electrodynamics in terms of cohomology theory and thus obtained a natural interpretation of the discreteness of charge. At the same time we showed that the laws were now in a form that is independent of whether or not there is an underlying space-time continuum.

In later papers we are going to extend this type of description so as to include the theory of gravitation and elementary particles. In doing this, we shall go into more detail as to how the necessary mathematics is to be developed.

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